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Third Semester B.E. Degree Examination, June/July 2023
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define Tautology and Contradiction. Prove that $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology. **(05 Marks)**

b. Prove the following logical equivalences using laws of logic :

(i) $(p \vee q) \wedge [\sim (\sim p \wedge q)] \Leftrightarrow p$.

(ii) $\sim [\sim \{(p \vee q) \wedge r\} \vee \sim q] \Leftrightarrow q \wedge r$ **(05 Marks)**

c. Let the universe comprises all real numbers. The open statement P(x), Q(x), R(x), S(x) are given by,

$P(x) : x \geq 0$ $R(x) : x^2 - 3x - 4 = 0$

$Q(x) : x^2 \geq 0$ $S(x) : x^2 - 3 > 0$

Evaluate

(i) $\exists x[P(x) \wedge R(x)]$

(ii) $\forall x[P(x) \to Q(x)]$

(iii) $\forall x[Q(x) \to S(x)]$ **(06 Marks)**

OR

2 a. Define Converse, Inverse and Contra positive. Write dual, converse, inverse and contrapositive of the following statement:

"If a triangle is not isosceles, then it is not equilateral". **(06 Marks)**

b. Simplify the following compound propositions by using laws of logic :

$p \to (q \to r) \equiv (p \wedge q) \to r$ **(04 Marks)**

c. Test the validity of the following argument :

If I study, I will not fail in the exam

If I do not watch TV in the evenings, I will study

I failed in the exam

$\therefore$ I must have watched TV in the evenings **(06 Marks)**

Module-2

3 a. Prove by mathematical induction that $1^2 + 3^2 + 5^2 + \dots\dots\dots + (2n-1)^2 = \dfrac{n(2n-1)(2n+1)}{3}$ for all integers $n \geq 1$. **(05 Marks)**

b. A sequence $\{a_n\}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. Find a_n in explicit form. **(05 Marks)**

c. How many arrangements are there for all letters in the word MASSASAUGA? In how many of these arrangements all the vowels are adjacent? **(06 Marks)**

OR

4 a. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(06 Marks)**

b. Evaluate the number of ways can 10 identical dimes be distributed among five children if, (i) there are no restrictions (ii) Each child gets at least one dime. (iii) The oldest child gets at least two dimes. **(06 Marks)**

c. Explain Binomial theorem. **(04 Marks)**

Module-3

5 a. List and explain types of functions with examples. **(05 Marks)**

b. Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only iff "a is a multiple of b". Represent the relation R as a matrix, draw its digraph and find in-degrees, out-degress of each vertex. **(05 Marks)**

c. If R be a relation on the set $A = \{1, 2, 3, 4, 6, 8, 12\}$ defined by aRb if a divides b then (i) prove that (A, R) is a POSET (ii) Draw the Hasse diagram of the (A, R). **(06 Marks)**

OR

6 a. Let $f : R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & \forall x > 0 \\ -3x + 1 & \forall x \le 0 \end{cases}$. Determine f(0), f(-1), $f\left(\frac{5}{3}\right)$, $f^{-1}(3)$ and $f^{-1}([5, -5])$. **(05 Marks)**

b. If $f : R \to R$ and $g : R \to R$ be functions defined by $f(x) = 3x + 7$, $g(x) = x(x^3 - 1)$, $\forall x \in R$, verify that f is one-to-one but g is not. **(05 Marks)**

c. Let N be the set of all natural numbers on $N \times N$ the relation R is defined as (a, b)R(c, d) if and only if $a + d = b + c$. Show that R is equivalence relation. **(06 Marks)**

Module-4

7 a. A student taking DMS examination is directed to answer 7 out of 10 questions. Determine the possible ways if,
   (i) There is no concern about the order.
   (ii) Must answer 3 questions from first 5 and four from last 5 questions. **(08 Marks)**

b. What do you mean by derangements? Evaluate d_5, d_6, d_9 **(08 Marks)**

OR

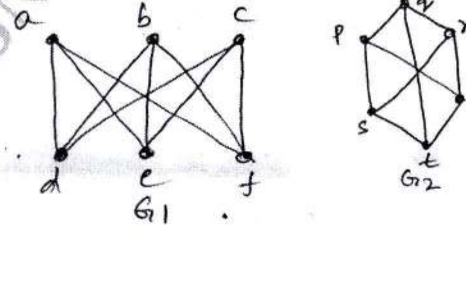
8 a. Calculate the number of positive integers n such that $1 \le n \le 100$ and n is not divisible by 2, 3 or 5. **(08 Marks)**

b. Find the rook polynomial for 3*3 board (place * at center) by using the expansion formula. **(08 Marks)**

Module-5

9 a. Explain the following with example :
   (i) Multigraph (ii) Regular graph
   (iii) Complement graph. (iv) Induced sub graph. **(05 Marks)**

b. Show that the following graphs are isomorphic :



(06 Marks)

c. Define prefix codes. Explain how to construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with a frequency of 20, 28, 4, 17, 12, 7 respectively. **(05 Marks)**

OR

10 a. For the graph shown below determine,
   (i) A walk from b to d that is not trail.
   (ii) A b to d trail that is not a path.
   (iii) A path from b to d.
   (iv) A circuit from b to b that is not cycle. **(05 Marks)**

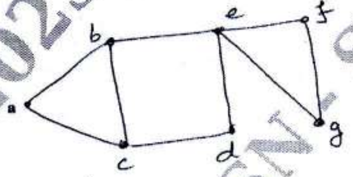


Fig. Q10 (a)

b. Define tree. Show that a tree with n vertices has n – 1 edges. **(05 Marks)**

c. What are prefix codes? Obtain the optimal prefix code for the message "ROAD IS GOOD". Indicate the code. **(06 Marks)**

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